[07-09-03-T10]

Order of the real numbers

Order Axioms

◆ **The real numbers satisfy the following axioms.**

O1 (trichotomy). $\forall x, \forall y, \forall z \in \mathbb{R}$, *exactly one* of the following is true

$x = y$ *or* $x < y$ *or* $y < x$

O2 (transitive). $\forall x, \forall y, \forall z \in \mathbb{R}$,

if $x < y$ *and* $y < z$, *then* $x < z$

O3 $\forall x, \forall y, \forall z \in \mathbb{R}$,

if $x < y$, *then* $x + z < y + z$

O4 $\forall x, \forall y, \forall z \in \mathbb{R}$,

if $0 < z$ *and* $x < y$, *then* $z \cdot x < z \cdot y$

Note: the symbol "$<$" is pronounced "less than". We read "$a < b$" as "a is less than b". To better understand the relation "less than", imagine the real number line. When we say "$a < b$", we assert that a lies to the left of b on the real number line.

◆ **Definitions.**

$x > y$ *means* $y < x$
$x \le y$ *means* $x < y$ **or** $x = y$
$x \ge y$ *means* $x > y$ **or** $x = y$

■ Discussion

Axiom **O3** allows us to add across an inequality. Axiom **O4** tells us the circumstances in which multiplication preserves the inequality. Note that **O4** concerns only the circumstance when we multiply by a number greater than zero.